Applied Mathematics in Engineering, Management and Technology 2 (4) 2014: 267-271

www.amiemt-journal.com

Adomian Decomposition Method for solving Fuzzy Nonlinear Mixed Volterra-Fredholm Integral Equation

Hadis Seihei¹, Katira Ghorbanpoor², Maryam Rezaii³

¹Department of Mathematics, Andimeshk Branch Applied and Sciences University of municipality, Andimeshk, Iran
²Department of Mathematics, Andimeshk Branch, Islamic Azad University, Andimeshk, Iran
³Department of Mathematics, Arak Branch, Islamic Azad University, Arak, Iran

¹E-mail address: hadis_seihei@yahoo.com
²E-mail address: katira_ghorbanpor@yahoo.com
³E-mail address: mryamrezaii@yahoo.com

Abstract:

In this paper, we use of Adomian decomposition method (ADM) for solving fuzzy nonlinear mixed volterra-fredholm integral equation of the second kind. We convert a fuzzy nonlinear mixed volterra-fredholm integral equation to a nonlinear system of mixed volterra-fredholm integral equation in crisp case. We can use of ADM and find the approximation solution of the system and hence obtain an approximation for fuzzy solution of the fuzzy nonlinear mixed volterra-fredholm integral equation of the second kind. The proposed method is illustrated by solving a numerical example.

Keywords: fuzzy nonlinear mixed volterra-fredholm integral equation; Adomian decomposition method.

1. Introduction

The concept of fuzzy numbers and arithmetic operations with this numbers were first introduced and investigated by Zadeh [24] and others. S. Abbasbandy et al. [1] suggested a new Numerical method for solving linear Fredholm fuzzy integral equations of the second kind. The topics of fuzzy integral equations (FIE) which attracted growing interest for some time, in particular in relation to fuzzy control, have been developed in recent years. Goetschel and Vaxman [15] suggested a new approach, they represented the fuzzy number as a parameterized triple and then embedded the set of fuzzy numbers into a topological vector space. The establishment of the embedding banach space and its induced metric over its subset of fuzzy numbers led to immediate applications such as fuzzy least square [7,8,16,20], fuzzy linear system [4,5], fuzzy eigenvalues and eigenvectors [5,13]. Further applications such as solving fuzzy integral equations requested appropriated and applicable definitions of the fuzzy function and the fuzzy integral of fuzzy function. Congxin and Ming [21] represent the first applications of fuzzy integration. Numerical solution of linear fuzzy Fredholm integral equations of the second kind by Adomian method was introduced by Babolian et al. [3]. There are several research papers about obtaining the numerical integration of fuzzy-valued functions and solving fuzzy Volterra and Fredholm integral equations [9,10,11]. We think that, this paper can be used to convey to student the idea that the ADM is a powerful tool for solving fuzzy nonlinear mixed volterra-fredholm integral equations of the second kind. An example of fuzzy nonlinear mixed volterra-fredholm integral equation is the obtained results that the ADM is very effective and simple.

2. Preliminaries

In this section the most basic notation used in fuzzy calculus are introduced. We start by defining a fuzzy number.

Definition 1. [1] A fuzzy number is a map \( u: \mathbb{R} \rightarrow I = [0,1] \) which satisfies

(i) \( u \) is upper semi-continuous.

(ii) \( u(x) = 0 \) Outside some interval \([c, d] \subset \mathbb{R}\).

(iii) There exist real numbers \( a, b \) such that \( c \leq a \leq b \leq d \) where
The set of all such fuzzy numbers is represented by $\mathbb{E}$. For arbitrary $u = (u(r), \overline{u}(r))$, $v = (v(r), \overline{v}(r))$ and $k \in \mathbb{R}$ we define addition and multiplication by $k$ as:

$$(u + v)(r) = (u(r) + v(r)), \quad (ku)(r) = ku(r), \quad (k\overline{u})(r) = k\overline{u}(r)$$

Definition 2. For arbitrary fuzzy numbers $u, v \in \mathbb{E}$, we use the distance [16]

$$D(u, v) = \sup_{0 \leq r \leq 1} \max \{|\overline{u}(r) - \overline{v}(r)|, |u(r) - v(r)|\}$$

And it is shown that $(\mathbb{E}, D)$ is a complete metric space [24].

Definition 3. [11] Let $f : \mathbb{R} \to \mathbb{E}$ be a fuzzy function. If for arbitrary fixed $x_0 \in \mathbb{R}^2$ and $\varepsilon > 0$, exists $\delta > 0$ suchthat $0 < |t - t_0| < \delta \Rightarrow D(f(t), f(t_0)) < \varepsilon$ $f$ is said to be continuous.

Definition 4. [15, 16] Let $f : [a, b] \to \mathbb{E}$, for each partition $p = \{t_0, t_1, ..., t_n\}$ of $[a, b]$ and for arbitrary $\xi_i \in [t_{i-1}, t_i]$ $1 \leq i \leq n$, suppose

$$R_p = \sum_{i=1}^{n} f(\xi_i)(t_i - t_{i-1}),$$

$$\Delta = \max\{|t_i - t_{i-1}|, i = 1, ..., n\}.$$ The definite integral of $f(t)$ over $[a, b]$ is

$$\int_{a}^{b} f(s, t) \, ds \, dt = \lim_{\Delta \to 0} R_p$$

Provided that this limit exists in the metric $D$.If the fuzzy function $f(t)$ is continuous in the metric $D$, its definite integral exists, and also,

$$\int_{a}^{b} f(t; r) dt = \int_{a}^{b} f(t; r) dt$$

$$\int_{a}^{b} f(t; r) dt = \int_{a}^{b} f(t; r) dt.$$

3. Fuzzy nonlinear mixed volterra-fredholm integral equations

The mixed volterra-fredholm integral equation of the second kind is an integral equation of the form [22]:

$$u(x; r) = f(x; r) + \lambda \int_{0}^{x} \int_{a}^{b} k(s, t) u(t; r) dt \, ds, \quad (1)$$

In this study we survey the fuzzy nonlinear mixed volterra-fredholm integral equation of the second kind is as follows:

$$u(x; r) = f(x; r) + \lambda \int_{0}^{x} \int_{a}^{b} k(s, t) G(t, u(t; r)) dt \, ds \quad (2)$$

$$a \leq t \leq x \leq b, 0 \leq s \leq x,$$

Where $\lambda > 0$, $f(x, r)$ is a fuzzy function, $k(s, t)$ is an arbitrary kernel function, and $0 \leq r \leq 1$. It is interesting to note that (1) contains mixed Volterra and Fredholm integral equations.

$$u(x; r) = f(x; r) + \lambda \int_{0}^{x} \int_{a}^{b} k(s, t) G(t, u(t; r)) dt \, ds \quad (3)$$

$$\overline{u}(x; r) = \overline{f}(x; r) + \lambda \int_{0}^{x} \int_{a}^{b} k(s, t) G(t, u(t; r)) dt \, ds$$

Let for $a \leq t \leq b$, we have:
Using ADM, we approximate the solution of the equation as an infinite series solution. In the special case of fuzzy mixed Volterra-Fredholm integral equations, we can write

\begin{align}
H_1(t, u, \overline{u}) &= \min \{G(t, \beta)| u(t; r) \leq \beta \leq \overline{u}(t; r)\} \\
H_2(t, u, \overline{u}) &= \max \{G(t, \beta)| u(t; r) \leq \beta \leq \overline{u}(t; r)\}
\end{align}

Then

\begin{align}
k(s; t)G(t, u(t; r)) &= \begin{cases} k(s, t)H_1(t, u, \overline{u}), & k(x, t) \geq 0 \\
k(s, t)H_2(t, u, \overline{u}), & k(x, t) < 0 \end{cases} \\
k(s; t)G(t, u(t; r)) &= \begin{cases} k(s, t)H_1(t, u, \overline{u}), & k(x, t) \geq 0 \\
k(s, t)H_2(t, u, \overline{u}), & k(x, t) < 0 \end{cases}
\end{align}

### 3.1. Adomian decomposition method

The ADM assume an infinite series solution for the unknown functions \([u, \overline{u}]\), given by

\begin{align}
\overline{u}(x; r) &= \sum_{i=0}^{\infty}\overline{u}_i(x), \quad \overline{u}(x; r) = \sum_{i=0}^{\infty}\overline{u}_i(x), \\
G(t, u(t); \overline{u}(t)), & \text{ into an infinite series of polynomials given by} \\
G(t, u(t)) &= \sum_{i=0}^{n} A_n(x), \quad G(t, \overline{u}(t)) = \sum_{i=0}^{n}\overline{A}_n(x).
\end{align}

Where \(A_n = [A_n, \overline{A}_n]; n \geq 0\), are the so-called Adomian polynomials defined by:

\begin{align}
A_n = \frac{1}{n!} \left[ \frac{d^n}{dt^n} \left( \sum_{i=0}^{n} \frac{d^i u(x)}{dx^i} \right) \right]_{t=0}, \\
\overline{A}_n = \frac{1}{n!} \left[ \frac{d^n}{dt^n} \left( \sum_{i=0}^{n} \frac{d^i \overline{u}(x)}{dx^i} \right) \right]_{t=0}.
\end{align}

Substituting (3) and (4) into (2), we get:

\begin{align}
\overline{u}_0 &= f(x; r), \\
\overline{u}_1 &= \int_a^x \int_a^b k(s, t)A_0 dt \, ds \\
\overline{u}_{n+1} &= \int_a^x \int_a^b k(s, t)A_n dt \, ds, \quad n \geq 0
\end{align}

And

\begin{align}
\overline{u}_0 &= \overline{f}(x; r), \\
\overline{u}_1 &= \int_a^x \int_a^b \overline{k}(s, t)A_0 dt \, ds \\
\overline{u}_{n+1} &= \int_a^x \int_a^b \overline{k}(s, t)A_n dt \, ds, \quad n \geq 0
\end{align}

We approximate \(u(x, r) = \{u(x; r), \overline{u}(x; r)\}\) by

\begin{align}
\omega_n &= \sum_{i=0}^{n} \overline{u}_i(x, r), \\
\overline{\omega}_n &= \sum_{i=0}^{n} \overline{u}_i(x, r)
\end{align}

Where, \(\lim_{n \to \infty} \omega_n = u(x, r), \lim_{n \to \infty} \overline{\omega}_n = \overline{u}(x, r)\).

### 4. Numerical Example

**Example 1.** Consider the following fuzzy mixed Volterra-Fredholm integral equation:

\begin{align}
u(x; r) &= f(x; r) + \int_0^x \int_0^1 t u^2(t; r) dt \, ds, \\
f(x; r) &= [r, 2-r], a \leq t \leq x \leq b, 0 \leq s \leq x,
\end{align}

Using ADM, we have:

\begin{align}
\overline{u}_0(x; r) &= r, \\
\overline{u}_1(x; r) &= \int_0^x \int_0^1 t \overline{u}_0^2(t; r) dt \, ds = \frac{1}{2} x r^2,
\end{align}

269
\[
\begin{align*}
\overline{u}_2(x; r) &= \int_0^x \int_0^1 t \left(2\overline{u}_0(t;r)\overline{u}_1(t;r)\right) dt \, ds = \frac{1}{3} xr^3, \\
\overline{u}_0(x; r) &= (2 - r), \\
\overline{u}_1(x; r) &= \int_0^x \int_0^1 t \, \overline{u}_0^2(t;r) dt \, ds = \frac{1}{2} x(2 - r)^2, \\
\overline{u}_2(x; r) &= \int_0^x \int_0^1 3x \left(2\overline{u}_0(t;r)\overline{u}_1(t;r)\right) dt \, ds = \frac{1}{3} x(2 - r)^3, 
\end{align*}
\]

Then, we obtain:
\[
\begin{align*}
\underline{u}(x; r) &= r + \frac{1}{2} x r^2 + \frac{1}{3} x r^3 + \cdots \\
\overline{u}(x; r) &= (2 - r) + \frac{1}{2} x(2 - r)^2 + \frac{1}{3} x(2 - r)^3 + \cdots
\end{align*}
\]

(Fig.1. Solution of Example 1, \(x \in [0, 0.5]\))

5. Conclusion

In this paper, we solved successfully the fuzzy nonlinear mixed volterra-fredholm integral equation using ADM. It is apparently seen that ADM is powerful in finding approximate solution for such equations. It does not need the difficult calculation.

References